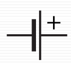
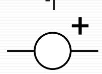
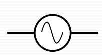
Electronic Elements and Circuits

Voltage

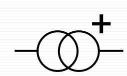
- Potential Energy from Charge Attraction
- Separation of Charge results in Stored Energy

- Electrical Potential energy is Measured in Volts (V) whose units are Joules/Coulomb
- 1 eV = 1.6 X 10E-19 Joules
- Voltage is sometimes called, "Electromotive Force" or *e.m.f.*
- The notation for charge is Q

Voltage Sources

- Batteries store electrical potential energy by chemically separating ions
- Salts separated across semi permeable membranes may be used as "batteries."
  - Symbol for a battery: 
  - Generic Voltage Source: 
  - Time Varying Voltage Source: 

Current

- Electrical Kinetic Energy is called Current
- Current is the motion of charge
- The Electrical Engineers symbol for current is *i* (*).
- Current Flows "through" conductors
- Current is therefore *dQ/dt*
- The Unit of Current is "Amperes" or amps.
  - Symbol for a current source: 

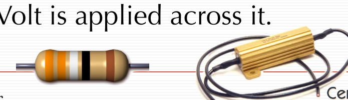
* *Hence, engineers use "j" to denote* $\sqrt{-1}$

Resistance

- Current flowing through a path experiences *Resistance*.
- Less current flow through higher resistance:
  - Ohm's Law: $i = V/R$
  - Larger resistance -> less current
- Energy is dissipated (lost) to that resistance
- As charge flows the stored energy is dissipated
- The *RATE* of Energy dissipation is measured in Watts (power, Joules/second)
- iV = (Joules/coulomb)(coulombs/s) = Joules/s.

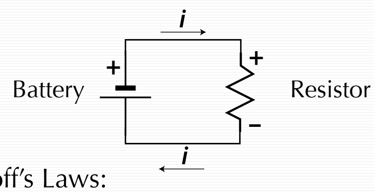
Resistance

- Insulators allow little or no current flow
- Conductors pass current easily.
  - conductor symbol:
- Typical "Resistors" range in values from about 1 Ohm to about 10E6 Ohm (10Megohm)
  - resistor symbol: 
- A 1 Ohm resistor allows 1 Ampere of current to flow when 1 Volt is applied across it.



Circuit

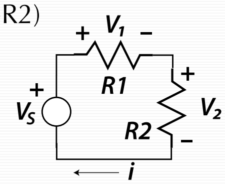
- Circuits always show the complete path for current flow



- Kirchhoff's Laws:
  - **KCL**: Current through any node adds to zero
    - any two terminal device is a node
  - **KVL**: Voltage around any loop adds to zero
  - Both laws are an expression of conservation of energy

Series Circuit - Voltage Divider

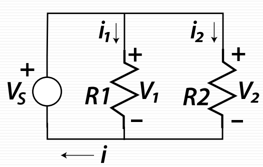
- In a series circuit **KVL** says that Vs = V1 + V2
- **KCL** says that i is the same in R1 and R2
- Ohms law states that $V_1 = i$R1 and $V_2 = i$R2
- Therefore: Vs = i(R1+R2)



- It follows that V_2 = Vs (R2/(R1+R2))

Parallel Circuit = Current Divider

- **KCL** says that $i = i_1 + i_2$
- **KVL** says that $V_1 = V_2$: Vs = i_1 R1 = i_2 R2
- The apparent resistance is: Vs/($i_1 + i_2$)



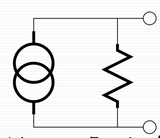
$$R_{\|} = \frac{V_S}{i_1 + i_2} = \frac{V_S}{\frac{V_S}{R1} + \frac{V_S}{R2}}$$

$$= \frac{1}{\frac{1}{R1} + \frac{1}{R2}}$$

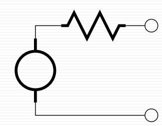
$$\boxed{= R1 \cdot R2 \big/ (R1 + R2)}$$

Norton and Thévenin Equivalent

- Real voltage and current sources have internal resistance



Norton Equivalent



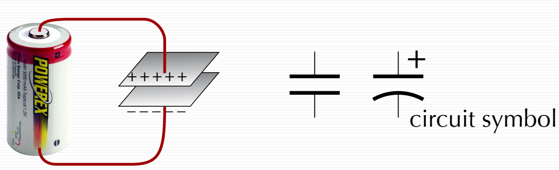
Thévenin Equivalent

In a real current source, as Load resistance increases, current drops

In a real voltage source, as Load resistance decreases, voltage drops

Capacitor

- When voltage is applied across an insulator charge moves onto the insulator.



- If the voltage source is removed, the separated charge stores potential energy
- Capacitance measures the amount of energy stored by separated charge: $C = Q/V$
- Capacitance is measured in Farads

Capacitor (cont'd)

- If charge is applied to one side of the capacitor, equal and opposite charge will move to the other side.
- This results in a net current "through" the capacitor.

$$Q = CV$$

$$\frac{dQ}{dt} = i = C\frac{dV}{dt}$$

- This appears similar to Ohm's law.

Laplace Transform

- Note that: $d(Ae^{st}) = sAe^{st}$
- Finding the derivative of a function of the form Ae^{st} is like multiplying by s
- Finding the integral is like dividing by s
- Applying the Laplace transform typically reduces differential equations to simple algebra.

Capacitors and Sinusoids

- Let: $V(t) = A\cos(\omega t)$:
- For a capacitor:

$$i_C = C\frac{dv}{dt} = -\omega CA\sin(\omega t)$$

$$-\omega CA\sin(\omega t) = \omega CA\cos(\omega t - 90°)$$

$$\frac{V}{i_C} = \frac{A\cos(\omega t)}{\omega CA\cos(\omega t - 90°)} = \frac{\cos(\omega t)}{\omega C\cos(\omega t - 90°)}$$

- A capacitor looks like a resistance whose magnitude goes as 1/ωC
- A capacitor introduces a 90° phase difference between current and Voltage.

Capacitors and Laplace

- Let $V(t) = Ae^{st}$

$$\frac{dV}{dt} = sAe^{st}$$

- Therefore $i = sCAe^{st}$

$$\frac{V}{i} = \frac{Ae^{st}}{sCAe^{st}} = \frac{1}{sC}.$$

- A capacitor acts like a resistance whose value depends on C and s!

Capacitor Demo

Wire

Aluminum Foil

$C = \varepsilon_0 A / D$

Typical Tape Thickness ~5E-5 m

$\varepsilon_0 \approx 8.854 \times 10^{-12} \quad F/m$

Laplace and Sinusoids

- Through Euler's formula with s=*i*ω (or *j*ω):

$$Ae^{st} = Ae^{j\omega t} = A(\cos(\omega t) + j\sin(\omega t))$$

- Letting: $V(t) = A\cos(\omega t)$

$$= \Re[Ae^{j\omega t}]$$

we see that: $i_C = sCAe^{st} = j\omega CA(\cos(\omega t) + j\sin(\omega t))$

$$= j\omega CA\cos(\omega t) - \omega CA\sin(\omega t)$$

- Whose real part is simply $i_C = -\omega CA\sin(\omega t)$ as before.

Impedance

- *Resistance* is the proportionality between constant current and constant Voltage.

$$V = iR$$

- *Impedance* is the ratio between time-varying Voltage and time-varying current.

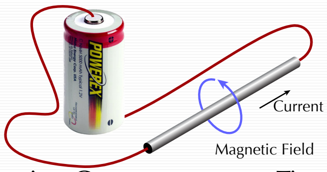
$$\mathbf{V = IZ}$$

Noting that **Z, I** and **V** may be complex values

- **Z** has a magnitude in Ohms, but may also include a phase.

Inductance

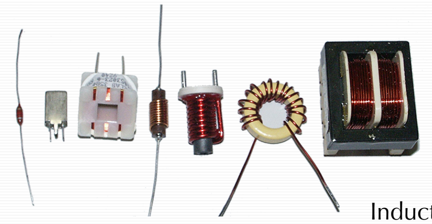
- Current creates a magnetic field about the conductor



- Time-varying Currents create a Time-Varying Field
- Time varying Magnetic Fields generate an e.m.f. that induces a time-varying current in conductors
- The e.m.f. is proportional the the rate of magnetic field change: $e.m.f. = k\frac{dB}{dt}$

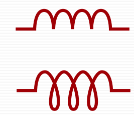
Inductors

- Commercial Inductors are simply coils of wire.



Inductor Circuit Symbol:

or

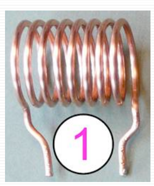


Inductors

- The magnetic field created by each loop of a coil is coupled to all of the other loops.
- In general, the magnetic field created by a time-varying current opposes the same current flow in the other coils
- The result is that:

$$V_L = L\frac{di}{dt}$$

where is the voltage across the inductor and L is the inductance value (in Henries).



Frequency Characteristics of Inductors

- Following the same reasoning as we used for a capacitor. Let: $i = Ae^{st}$, and $s = j\omega$

$$V = sLAe^{st}$$

- Thus

$$\boxed{\begin{aligned} i_L &= Ae^{st} \\ \frac{V_L}{i_L} &= L\frac{sAe^{st}}{Ae^{st}} \\ &= sL. \end{aligned}}$$

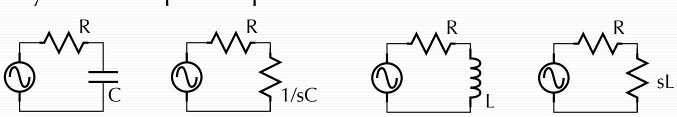
or:

$$\boxed{\begin{aligned} i_L &= \Re[Ae^{j\omega t}] = A\cos(\omega t) \\ \frac{d(i_L)}{dt} &= -\omega A\sin(\omega t) \\ \frac{V_L}{i_L} &= L\Re[\frac{-\omega A\sin(\omega t)}{A\cos(\omega t)}]. \end{aligned}}$$

- An inductor behaves like a resistor of magnitude sL that introduces a +90° phase shift.

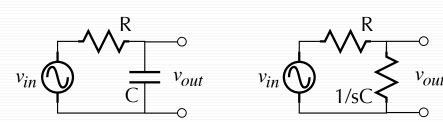
Complex Impedance

- Both Capacitors and Inductors have complex impedance: **V/I** is a complex quantity
- For a Capacitor, **V/I**=1/*sC*.
- For an Inductor, **V/I**=*sL*.
- In a circuit, we can replace all inductors and capacitors by their complex impedance:



- The circuits can then be analyzed with KVL and KCL

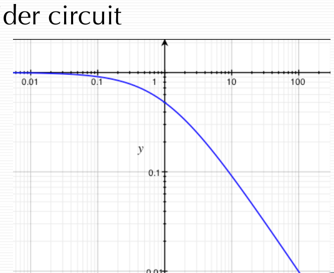
Example



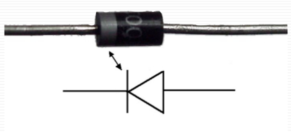
- This is just a Voltage Divider circuit

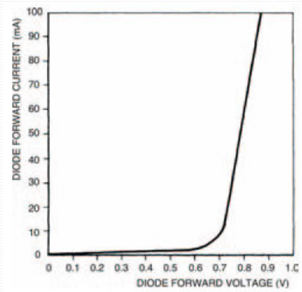
$$v_{out} = v_{in}\left(\frac{1/sC}{R + 1/sC}\right)$$

$$\frac{v_{out}}{v_{in}} = \frac{1}{sRC + 1}$$



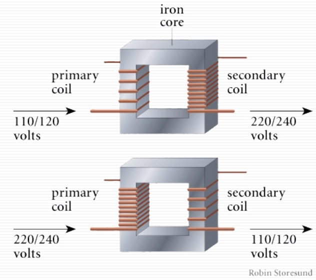
Diode

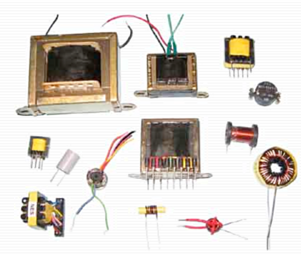




Making Signals Bigger

- Physiological signals are too small to observe directly
- Passive devices (transformer)





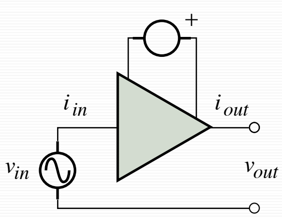
- Conservation of Energy: $v_{in} i_{in} = v_{out} i_{out}$

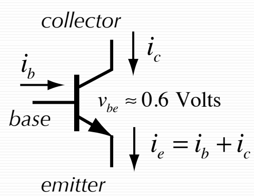
Amplifiers

- Generally: Total power is increased

$$v_{in} i_{in} < v_{out} i_{out}$$

- Amplifiers require an added source of energy



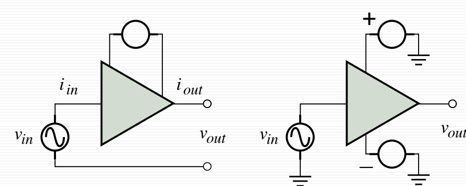


Transistor

Ground

- Ground is any selected node in a circuit

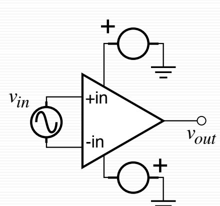
- Usually, ground is selected as either one side of the input signal or the power supply.
- All remaining Voltages are compared to Ground.



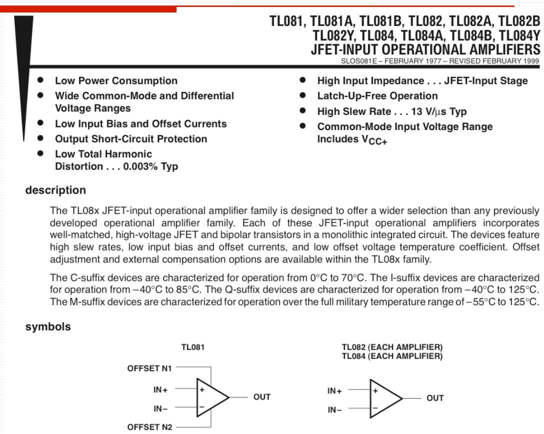
Operational Amplifier

- Ideal Op Amp
  - infinite gain
  - No current flows between +in and -in
- Real Op Amp
  - maximum output Voltage ≈ the power supply
  - gain > 1E4
  - input current << 1 μA

On the Op Amp:
+, $+in$, $v+$ are used equivalently
-, $-in$, $v-$ are used equivalently



Datasheet



TL081, TL081A, TL081B, TL082, TL082A, TL082B
TL082Y, TL084, TL084A, TL084B, TL084Y
JFET-INPUT OPERATIONAL AMPLIFIERS
SLOS081E – FEBRUARY 1977 – REVISED FEBRUARY 1999

- Low Power Consumption
- Wide Common-Mode and Differential Voltage Ranges
- Low Input Bias and Offset Currents
- Output Short-Circuit Protection
- Low Total Harmonic Distortion . . . 0.003% Typ
- High Input Impedance . . . JFET-Input Stage
- Latch-Up-Free Operation
- High Slew Rate . . . 13 V/μs Typ
- Common-Mode Input Voltage Range Includes V_{CC+}

description

The TL08x JFET-input operational amplifier family is designed to offer a wider selection than any previously developed operational amplifier family. Each of these JFET-input operational amplifiers incorporates well-matched, high-voltage JFET and bipolar transistors in a monolithic integrated circuit. The devices feature high slew rates, low input bias and offset currents, and low offset voltage temperature coefficient. Offset adjustment and external compensation options are available within the TL08x family.

The C-suffix devices are characterized for operation from 0°C to 70°C. The I-suffix devices are characterized for operation from –40°C to 85°C. The Q-suffix devices are characterized for operation from –40°C to 125°C. The M-suffix devices are characterized for operation over the full military temperature range of –55°C to 125°C.

symbols

TL081
OFFSET N1
IN+
IN–
OUT
OFFSET N2

TL082 (EACH AMPLIFIER)
TL084 (EACH AMPLIFIER)
IN+
IN–
OUT

Datasheet (cont'd)

electrical characteristics, $V_{CC\pm} = \pm 15$ V (unless otherwise noted)

	PARAMETER	TEST CONDITIONS	T_A†	TL081C TL082C TL084C MIN	TYP	MAX	TL081AC TL082AC TL084AC MIN	TYP	MAX	TL081BC TL082BC TL084BC MIN	TYP	MAX	TL081I TL082I TL084I MIN	TYP	MAX	UNIT
V_{IO}	Input offset voltage	$V_O = 0$, $R_S = 50\ \Omega$	25°C		3	15		3	6		2	3		3	6	mV
			Full range			20			7.5			5			9	
α_{VIO}	Temperature coefficient of input offset voltage	$V_O = 0$, $R_S = 50\ \Omega$	Full range		18			18			18			18		µV/°C
I_{IO}	Input offset current‡	$V_O = 0$	25°C		5	200		5	100		5	100		5	100	pA
			Full range			[illegible]			2			2			10	nA
I_{IB}	Input bias current‡	$V_O = 0$	25°C		30	400		30	200		30	200		30	200	pA
			Full range			[illegible]			7			7			20	nA
V_{ICR}	Common-mode input voltage range		25°C	±11	−12 to 15		±11	−12 to 15		±11	−12 to 15		±11	−12 to 15		V
V_{OM}	Maximum peak output voltage swing	$R_L = 10\ k\Omega$	25°C	±12	±13.5		±12	±13.5		±12	±13.5		±12	±13.5		V
		$R_L \geq 10\ k\Omega$	Full range	±12			±12			±12			±12			
		$R_L \geq 2\ k\Omega$		[illegible]	[illegible]		[illegible]	±12		±10	±12		±10	±12		
A_{VD}	Large-signal differential voltage amplification	$V_O = \pm 10$ V, $R_L \geq 2\ k\Omega$	25°C	25	200		[illegible]	200		50	200		50	200		V/mV
		$V_O = \pm 10$ V, $R_L \geq 2\ k\Omega$	Full range	[illegible]			25			25			25			
B_1	Unity-gain bandwidth		25°C		[illegible]			3			3			3		MHz
r_i	Input resistance		25°C		10^{12}			10^{12}			10^{12}			10^{12}		Ω
CMRR	Common-mode rejection ratio	$V_{IC} = V_{ICR}min$, $V_O = 0$, $R_S = 50\ \Omega$	25°C	70	86		75	86		75	86		75	86		dB
k_{SVR}	Supply voltage rejection ratio ($\Delta V_{CC\pm}/\Delta V_{IO}$)	$V_{CC} = \pm 15$ V to ± 9 V, $V_O = 0$, $R_S = 50\ \Omega$	25°C	70	86		80	86		80	86		80	86		dB
I_{CC}	Supply current (per amplifier)	$V_O = 0$, No load	25°C		1.4	2.8		1.4	2.8		1.4	2.8		1.4	2.8	mA
V_{O1}/V_{O2}	Crosstalk attenuation	$A_{VD} = 100$	25°C		120			120			120			120		dB

† All characteristics are measured under open-loop conditions with zero common-mode voltage unless otherwise specified. Full range for T_A is 0°C to 70°C for TL08_C, TL08_AC, TL08_BC and −40°C to 85°C for TL08_I.
‡ Input bias currents of a FET-input operational amplifier are normal junction reverse currents, which are temperature sensitive as shown in Figure 17. Pulse techniques must be used that maintain the junction temperature as close to the ambient temperature as possible.

Datasheet (cont'd)

**TL081, TL081A, TL081B, TL082, TL082A, TL082B
TL082Y, TL084, TL084A, TL084B, TL084Y
JFET-INPUT OPERATIONAL AMPLIFIERS**
SLOS081E – FEBRUARY 1977 – REVISED FEBRUARY 1999

absolute maximum ratings over operating free-air temperature range (unless otherwise noted)†

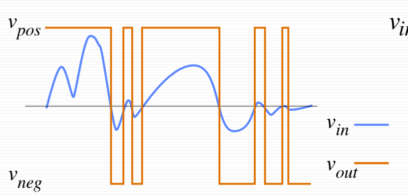
		TL08_C TL08_AC TL08_BC	TL08_I	TL084Q	TL08_M	UNIT
Supply voltage, V_{CC+} (see Note 1)		18	18	18	18	V
Supply voltage V_{CC-} (see Note 1)		−18	−18	−18	−18	V
Differential input voltage, V_{ID} (see Note 2)		±30	±30	±30	±30	V
Input voltage, V_I (see Notes 1 and 3)		±15	±15	±15	±15	V
Duration of output short circuit (see Note 4)		unlimited	unlimited	unlimited	unlimited	
Continuous total power dissipation		See Dissipation Rating Table				
Operating free-air temperature range, T_A		0 to 70	−40 to 85	−40 to 125	−55 to 125	°C
Storage temperature range, T_{stg}		−65 to 150	−65 to 150	−65 to 150	−65 to 150	°C
Case temperature for 60 seconds, T_C	FK package				260	°C
Lead temperature 1,6 mm (1/16 inch) from case for 60 seconds	J or JG package				300	°C
Lead temperature 1,6 mm (1/16 inch) from case for 10 seconds	D, N, P, or PW package	260	260	260		°C

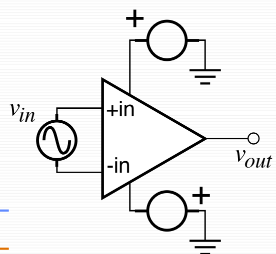
† Stresses beyond those listed under "absolute maximum ratings" may cause permanent damage to the device. These are stress ratings only, and functional operation of the device at these or any other conditions beyond those indicated under "recommended operating conditions" is not implied. Exposure to absolute-maximum-rated conditions for extended periods may affect device reliability.

NOTES: 1. All voltage values, except differential voltages, are with respect to the midpoint between V_{CC+} and V_{CC-}.
2. Differential voltages are at IN+ with respect to IN−.
3. The magnitude of the input voltage must never exceed the magnitude of the supply voltage or 15 V, whichever is less.
4. The output may be shorted to ground or to either supply. Temperature and/or supply voltages must be limited to ensure that the dissipation rating is not exceeded.

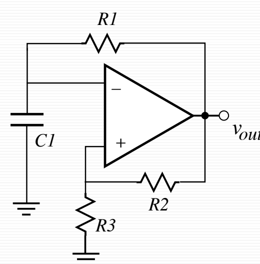
Op Amp Non-linear Operation

- "Open Loop" mode.
  - E.g., "Comparator"
  - If: +in>-in then $v_{out} \approx v_{pos}$
  - If: +in<-in then $v_{out} \approx v_{neg}$





Multivibrator

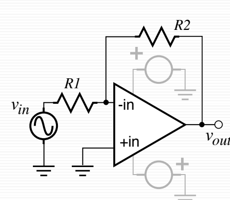


Linear Operation for Op Amps

- Negative Feedback
- +in ≈ -in
- $-V_{CC} < V_{out} < +V_{CC}$

- Voltage at inverting (*v–*, or *-in*) and non-inverting (*v+, or +in*) inputs is equal.
- No current flows between these inputs
- v_{out} is adjusted as needed for the above to be true.

Inverting Amplifier



In these slides, *-in* is the Voltage at the inverting input of the op amp (with respect to ground), and *+in* is the voltage at the non-inverting input.

In this circuit, negative feedback is used to ensure that *v–* and *v+* are kept equal. In this case, they are kept at ground.

$$v_{R1} = v_{in}$$

$$i_{R1} = \frac{v_{in}}{R1}$$

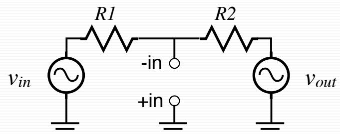
Because no current can flow between the inverting (–) and non-inverting (+) inputs to the op amp, the current through R2 must equal *i*R1. Therefore the Voltage across R2 must equal R2* *i*R1. This Voltage must therefore be sourced by the output of the op amp:

$$v_{out} = -i_{R1}R2$$

$$= -R2\frac{v_{in}}{R1}$$

$$\boxed{\frac{v_{out}}{v_{in}} = \frac{-R2}{R1}}$$

Inverting Amplifier Equivalent Circuit



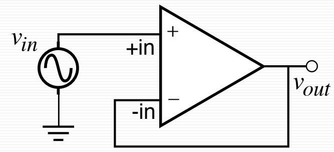
In an op amp, v_{out} is controlled by the difference between *-in* and *+in*. The output Voltage is fed back (*negative feedback*) to the *v*– input so that the (*+in – -in*) ≈ 0.

No current flows between *+in* and *-in* therefore, in this case, the current through R1 also goes through R2. The energy to supply that current is provided by the op amp (actually from its power supplies).

Notice the direction of the current through R2: when v_{in} is positive, v_{out} must be negative.

From the perspective of the input source, the op amp can be modeled as a resistor of value, R1.

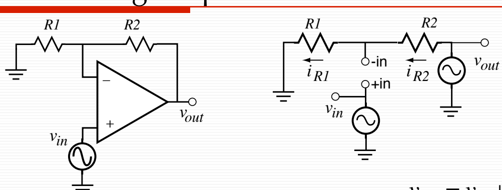
Voltage Follower



At first blush, this very common op amp circuit seems odd. After all, it is clear that if *-in* and *+in* are equal $v_{out} = v_{in}$.

What makes this useful, is that no matter what load v_{out} is connected to, the op amp ensures that no current flows into the *+in* input. The Voltage follower *isolates* the input source from the load driven by v_{out}. This means that the input source is not altered by driving a load. Essentially no current flows out of the input source (which therefore loses no energy).

Non-Inverting Amplifier



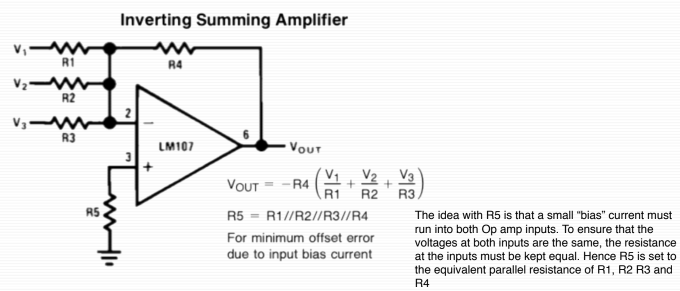
$$v_{out} = v_{in} + i_{R1}R2$$
$$= v_{in} + \frac{v_{in}}{R1}R2$$
$$= v_{in}\left(1 + \frac{R2}{R1}\right)$$

In this case the *-in* input is going to be set to v_{in} by the output Voltage source of the op amp. This means that v_{out} must be equal to the Voltage across R2, plus the Voltage across R1 (which is v_{in}).

$$\frac{v_{out}}{v_{in}} = 1 + \frac{R2}{R1} = \frac{R1 + R2}{R1}$$

If v_{in} is positive the current flows in the direction shown. This means that v_{out} also is positive.

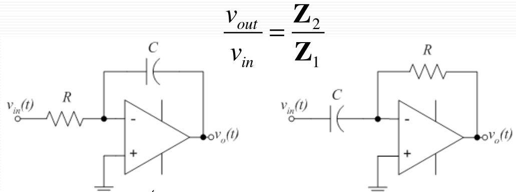
Inverting Summing Amplifier



The current through R4 is equal to the sum of the currents through R1, R2 and R3 (*KCL*).

Differentiator and Integrator

$$\frac{v_{out}}{v_{in}} = \frac{\mathbf{Z}_2}{\mathbf{Z}_1}$$



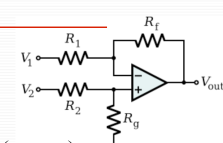
$$\frac{v_{out}}{v_{in}} = \frac{1/sC}{R} = \frac{1}{sRC}$$

$$\frac{v_{out}}{v_{in}} = \frac{R}{1/sC} = sRC$$

$$v_o(t) = \frac{1}{RC}\int_0^t v_{in}(t)dt + v_0(0)$$

$$v_0(t) = -RC\frac{dv_{in}(t)}{dt}$$

Difference Amplifier



A "difference amplifier" amplifies the *difference* in voltage between to points, *v*1 and *v*2, rejecting any Voltage they have in common.
The current through R1, *i*R1, is (*v*1–*v*+)/R1, and is the same as the current through R2, which is (*v*+–*v*out)/R2.
The Voltage divider at the non-inverting input ensures that: $v_+ = v_2\left(\frac{R_g}{R_2 + R_g}\right)$.

$$\frac{v_1 - v_+}{R_1} = \frac{v_+ - v_{out}}{R_f}$$

$$\frac{v_1 - v_2\left(\frac{R_g}{R_2 + R_g}\right)}{R_1} = \frac{v_2\left(\frac{R_g}{R_2 + R_g}\right) - v_{out}}{R_f}$$

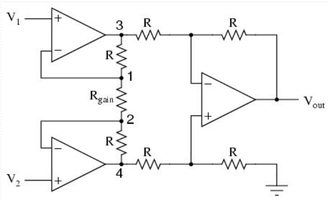
$$R_f v_1 - v_2\frac{R_f R_g}{R_2 + R_g} = v_2\frac{R_1 R_g}{R_2 + R_g} - R_1 v_{out}$$

$$R_1 v_{out} = v_2\frac{R_1 R_g}{R_2 + R_g} + v_2\frac{R_f R_g}{R_2 + R_g} - R_f v$$

$$v_{out} = v_2\frac{R_g}{R_2 + R_g} + v_2\frac{R_f R_g}{R_1(R_2 + R_g)} - v_1\frac{R_f}{R_1}$$
$$= v_2\left(\frac{R_1 R_g}{R_1(R_2 + R_g)} + \frac{R_f R_g}{R_1(R_2 + R_g)}\right) - v_1\frac{R_f}{R_1}$$
$$= v_2\left(\frac{R_g\left(R_1 + R_f\right)}{R_1(R_2 + R_g)}\right) - v_1\frac{R_f}{R_1}$$

If $R_1=R_2$ and $R_f=R_g$: $v_{out} = (v_2 - v_1)\frac{R_2}{R_1}$

Instrumentation Amplifier



An instrumentation amplifier is essentially a difference amplifier whose inputs are isolated from the source by Voltage followers. Virtually no current flows between v_1 and v_2.

Why? Because any difference in Voltage between the v_1 and v_2 terminals of the first op amps must be matched by the Voltage across the two $v-$ terminals. This appears across R3. The current to produce this drop must come through the two R2 resistors. If they are large that current will create a large Voltage across them.

Integrated Instrumentation Amplifier

FEATURES

- LOW OFFSET VOLTAGE: 50µV max
- LOW DRIFT: 0.25µV/°C max
- LOW INPUT BIAS CURRENT: 2nA max
- HIGH COMMON-MODE REJECTION: 115dB min
- INPUT OVER-VOLTAGE PROTECTION: ±40V
- WIDE SUPPLY RANGE: ±2.25 to ±18V
- LOW QUIESCENT CURRENT: 3mA max
- 8-PIN PLASTIC AND SOL-16

APPLICATIONS

- BRIDGE AMPLIFIER
- THERMOCOUPLE AMPLIFIER
- RTD SENSOR AMPLIFIER
- MEDICAL INSTRUMENTATION
- DATA ACQUISITION

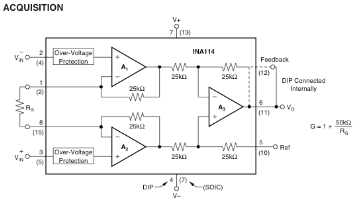
DESCRIPTION

The INA114 is a low cost, general purpose instrumentation amplifier offering excellent accuracy. Its versatile 3-op amp design and small size make it ideal for a wide range of applications.

A single external resistor sets any gain from 1 to 10,000. Internal input protection can withstand up to ±40V without damage.

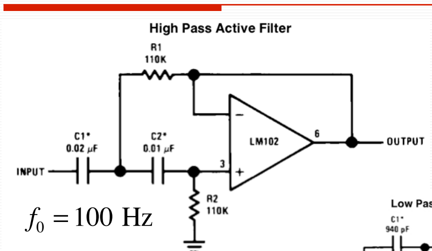
The INA114 is laser trimmed for very low offset voltage (50µV), drift (0.25µV/°C) and high common-mode rejection (115dB at G = 1000). It operates with power supplies as low as ±2.25V, allowing use in battery operated and single 5V supply systems. Quiescent current is 3mA maximum.

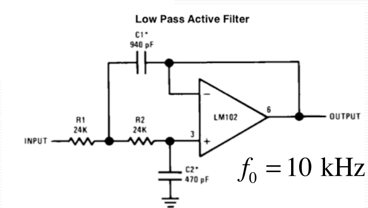
The INA114 is available in 8-pin plastic and SOL-16 surface-mount packages. Both are specified for the –40°C to +85°C temperature range.



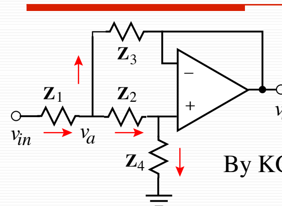
By making the amplifier on a single piece of silicon, the manufacturer can ensure that all of the resistors are matched precisely. In turn, this makes sure that common mode signals are heavily attenuated.

Second Order Filter





Second Order Filter Analysis



$$v_+ = v_- = v_{out} \quad \text{(op amp rules)}$$

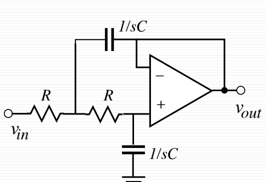
By KCL: $$\frac{(v_{in} - v_a)}{\mathbf{Z}_1} = \frac{(v_a - v_{out})}{\mathbf{Z}_2} + \frac{(v_a - v_{out})}{\mathbf{Z}_3}, \text{ and}$$

$$\frac{(v_a - v_{out})}{\mathbf{Z}_2} = \frac{v_{out}}{\mathbf{Z}_4}$$

Although the algebra is tedious, these can be solved for v_{out}/v_{in}:

$$\frac{v_{out}}{v_{in}} = \frac{\mathbf{Z}_3\mathbf{Z}_4}{\mathbf{Z}_3\mathbf{Z}_4 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_2}$$

Low Pass Filter



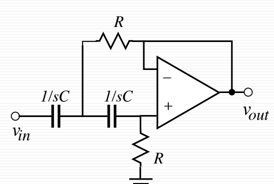
$$\frac{v_{out}}{v_{in}} = \frac{\mathbf{Z}_3\mathbf{Z}_4}{\mathbf{Z}_3\mathbf{Z}_4 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_2}$$

$$\mathbf{Z}_1 = \mathbf{Z}_2 = R, \quad \mathbf{Z}_3 = \mathbf{Z}_4 = \frac{1}{sC}$$

$$\frac{v_{out}}{v_{in}} = \frac{1/s^2C^2}{1/s^2C^2 + R/sC + R/sC + R^2}$$

$$= \frac{1}{1 + 2sRC + s^2R^2C^2} = \frac{1}{(1 + sRC)^2}$$

High Pass Filter



$$\frac{v_{out}}{v_{in}} = \frac{\mathbf{Z}_3\mathbf{Z}_4}{\mathbf{Z}_3\mathbf{Z}_4 + \mathbf{Z}_2\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_1\mathbf{Z}_2}$$

$$\mathbf{Z}_1 = \mathbf{Z}_2 = \frac{1}{sC}, \quad \mathbf{Z}_3 = \mathbf{Z}_4 = R$$

$$\frac{v_{out}}{v_{in}} = \frac{R^2}{R^2 + R/sC + R/sC + 1/s^2C^2}$$

$$= \frac{R^2s^2C^2}{1 + 2sRC + s^2R^2C^2} = \frac{(sRC)^2}{(1 + sRC)^2}$$

First and Second Order Filters